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Gravitationally Stabilized Nucleons in the Theory of the Generalized Gravitational Potential

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Abstract

From the field equation in ψ of the author's general-relativistic scalar field theory in the complex Weyl space an infinite set of Yukawa type equations, representing potential fields, arising from zero rest mass particles is obtained. The simplest of these equations is solved and is used in a nucleon model as the origin of a gravitationally stabilized nucleon. This procedure leads directly to an expression that predicts the representative nucleon radius of 1·21 fm.

1. *Relative Scalar Fields for Zero Rest Mass Particles*

If a function of the coordinates $\psi = \psi(\chi^\mu)$ transforms, under a coordinate transformation, as

$$\bar{\psi} = J^w \psi \tag{1.1}$$

where J is the Jacobian of the transformation and w is a constant, then ψ is called a relative scalar of weight w. If $w = 0$, ψ is called an absolute scalar and if $w = 1$, ψ is given the name 'scalar density' or 'pseudoscalar' (Thomas, 1965; Sokolnikoff, 1964; Møller, 1952). In the general relativistic scalar field theory in the complex Weyl space (Cherry, 1971a) the field equation for such scalars has the form

$$\eta^{\rho\sigma}\psi_{:\rho:\sigma} + \left[\frac{\eta^{\rho\sigma}R_{\rho\sigma}}{6} - \kappa^2\right]\psi = 0 \tag{1.2}$$

where the colon notation specifies covariant differentiation for relative tensors, $R_{\rho\sigma}$ is the Ricci tensor taken in the complex Weyl space and $\kappa = m_0 c/\hbar$. In this discussion we will consider only particles with zero rest mass so that we have $\kappa \to 0$. In addition, we note that the complex Weyl space reduces to the real Riemann space with the vanishing of the components of the electromagnetic 4-potential ϕ_λ, that appear in $\psi_{:\rho:\sigma}$ and $R_{\rho\sigma}$ of equation (1.2) (Cherry, 1971a). As a consequence of letting $\phi_\lambda = 0$

we have $\eta^{\rho\sigma} \to g^{\rho\sigma}$, where $g^{\rho\sigma}$ is the metric tensor of the Riemann space. In this latter space Einstein's field equations hold so that we have

$$R_{\rho\sigma} = 0 \tag{1.3}$$

In view of the zero rest mass specification and equation (1.3) the second term of equation (1.2) vanishes and we therefore seek expressions of the form

$$g^{\rho\sigma} \psi_{:\rho:\sigma} = g^{\rho\sigma}[\psi_{,\rho,\sigma} - w\psi_{,\sigma} \Gamma^{\alpha}_{\rho\alpha} - w\psi \Gamma^{\alpha}_{\rho\alpha,\sigma} - \psi_{,\alpha} \Gamma^{\alpha}_{\rho\sigma} + w\psi \Gamma^{\alpha}_{\rho\sigma} \Gamma^{\beta}_{\beta\alpha} - w\psi_{,\rho} \Gamma^{\alpha}_{\sigma\alpha} + w^2 \psi \Gamma^{\alpha}_{\alpha\sigma} \Gamma^{\beta}_{\beta\rho}] = 0 \tag{1.4}$$

Restricting our attention to problems with spherical symmetry we may adopt the line element

$$ds^2 = -g_{00} c^2 dt^2 + g_{11} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \tag{1.5}$$

which with equation (1.3) gives the 'Schwarzschild' solution

$$g_{00} = \frac{1}{g_{11}} = 1 - \frac{b}{r}, \qquad b \geqslant 0 \tag{1.6}$$

where b is a constant of integration. We now consider the Yukawa specification $\psi = \psi(r)$ in which case equation (1.4) with equation (1.6) reduces to

$$[(r^2 - br)\psi']' - 4w(r - b)\psi' - w[1 - 4w(1 - b/r)]\psi = 0 \tag{1.7}$$

where the primes denote differentations with respect to r. Equation (1.7) includes all of the scalar equations in this theory that are applicable in Yukawa theory for zero rest mass particles with spherical symmetry. In particular for ψ as an absolute scalar ($w = 0$) we have

$$[(r^2 - br)\psi']' = 0, \qquad w = 0 \tag{1.8}$$

This equation differs from the expression obtained from Klein–Gordon theory by the term $-br$. If there were no gravitational field ($b = 0$) we would obtain the usual Coulomb solution for ψ in equation (1.8).

2. *Gravitationally Stabilized Nucleons*

According to the Yukawa prescription equation (1.8) implies that there exists an interaction potential

$$V(r) = \frac{a_1}{b} \ln\left(1 - \frac{b}{r}\right) + a_2 \tag{2.1a}$$

arising from particles with zero rest mass. We note from equation (2.1a) that

$$V(r) = -\frac{a_1}{b}\left[\frac{b}{r} + \frac{1}{2}\left(\frac{b}{r}\right)^2 + \frac{1}{3}\left(\frac{b}{r}\right)^3 + \dots\right] + a_2, \qquad r > b \tag{2.2}$$

and that for $r \sim \infty$

$$V_\infty \approx -\frac{a_1}{r} + a_2 \tag{2.3}$$

and setting the potential equal to zero for $r = \infty$, implies $a_2 = 0$. We, therefore, finally have

$$V(r) = \frac{a_1}{b} \ln\left(1 - \frac{b}{r}\right) \tag{2.1b}$$

Equation (2.1b) can have physical meaning, of course, only after the integration constants (b, a_1) are identified. In previous work (Cherry, 1971a; Cherry, 1971b) on the generalized gravitational potential we established that $b = r_0 A^{1/3}$ where r_0 is the nucleon core radius and A is the mass number. We now specify that a_1 is related to the pion-nucleon coupling constant by

$$a_1 = g_0 \frac{m_\pi}{m_n} \tag{2.4a}$$

where g_0 is the strong interaction coupling constant such that

$$g_0^2 \approx 15\hbar c \tag{2.4b}$$

and where m_π and m_n are the pion and nucleon rest masses, respectively. If equation (2.1b) together with the identification of b and equations (2.4a, b), is now assumed to represent the gravitational field responsible for nucleon stability and that

$$E = m_n c^2 = \tfrac{1}{2} \int_R^\infty \mathscr{E}^2 \, d\tau \tag{2.5}$$

where the intensity $\mathscr{E} = -dV/dr$ and R is some nucleon radius, we obtain

$$\begin{aligned} R &= r_0 + \left(g_0 \frac{m_\pi}{m_n}\right)^2 \frac{\lambda_n}{\hbar c} \\ &\approx r_0 + 15\lambda_n \left(\frac{m_\pi}{m_n}\right)^2, \qquad \lambda_n = \frac{h}{m_n c}. \end{aligned} \tag{2.6a}$$

The value for r_0 was previously determined from this theory when applied to the pionic atom (Cherry, 1971b) where it was found that for best fit with experimental data that $r_0 = 0{\cdot}78$ fm. Using this value for r_0 we find from equation (2.6a) that $R \approx 1{\cdot}21$ fm. This value for the nucleon radius is comparable to the 'electromagnetic' radius (Hofstadter, 1956), or 'nuclear radius constant' (DeBenedetti, 1964) obtained from electron and μ-meson scattering experiments, and is also comparable to the 'Coulomb radius constant' (Marmier & Sheldon, 1969) obtained from the Coulomb term of the nuclear binding energy formula directly, as well as those results from mirror nuclei.

Alternately, we could have obtained an expression predicting the nucleon mass, namely,

$$m_n = \left[\frac{2\pi}{R - r_0}\left(\frac{g_0 m_\pi}{c}\right)^2\right]^{1/3} \tag{2.6b}$$

provided the experimental determination of each constant on the right side of this equation does not require explicit use of the nucleon mass. Precision in the determination of m_n in this way is at present, of course, quite limited. The use of equation (2.5) implies, that the rest mass energy of the nucleon is equivalent to the energy required to disassemble the nucleon mass from a spherical shell of radius R to infinity, in a potential field given by equation (2.1b). In a more refined description of nucleon structure where the mass is distributed throughout the nucleon volume use of equations (2.1b) and (2.5) would lead to larger values of the nucleon mass than that obtained with equation (2.6b). This increased mass is compensated, however, by the equivalent mass arising from the energy of nucleon spin and Coulomb repulsion, that have not been taken into account with the use of equation (2.5). It is finally noted from the range of the static field of equation (2.1b) that nucleons can be formed without the nucleon constituents first assuming nuclear proximity.

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